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On Magnetic Dynamos in Thin Accretion Disks Around Compact and Young Stars

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A variety of geometrically thin accretion disks commonly associated with such astronomical objects as X-ray binaries, cataclysmic variables, and protostars are likely to be seats of MHD dynamo actions. Thin disk geometry and the particular physical environment make accretion disk dynamos different from stellar, planetary, or even galactic dynamos. We discuss those particular features of disk dynamos with emphasis on the difference between protoplanetary disk dynamos and those associated with compact stars. We then describe normal mode solutions for thin disk dynamos and discuss implications for the dynamical behavior of dynamo-magnetized accretion disks.

1 INTRODUCTION

It is widely appreciated that magnetic fields can play an important role in accretion disk dynamics. Shakura & Sunyaev (1973), in their well known paper, pointed to magnetic fields as the source of a viscous couple necessary for the accretion to take place. Disk magnetic fields have also been invoked to explain spectra of compact X-ray sources, as a source of coronal heating, and as a source of wind production. In the context of the Solar nebula, which is widely assumed to represent a typical protoplanetary disk, the existence of a magnetic field is inferred from the residual magnetization of primitive meteorites, which are assumed to owe their magnetization to nebular magnetic fields. However, in a typical accretion disk, the timescale for ohmic dissipation is much smaller than the typical radial infall time, thus it is difficult to see how any magnetic field contained in the gas that falls onto the disk can persist long enough to be dynamically or otherwise important, unless it is regenerated by a dynamo cycle. The particular dynamo mechanism

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	Accretion Disks Around Compact Stars	Protoplanetary Accretion Disks
radius	10^9 cm	10^{15} cm
thickness	10^6 cm	10^{12}–10^{13} cm
temperature	10^6–10^7 K	10^2–10^3 K
ionization	thermal	cosmic rays
R_m	10^9–10^{11}	10–10^3
t_{diff}	1–10^3 years	10–10^3 years

Table 1. Typical parameters of disks around compact stars (taken here to be a black hole of $10M_\odot$) and protoplanetary disks (taken here to be a disk around a T Tauri star of $1M_\odot$). R_m denotes the magnetic Reynolds number, t_{diff} denotes the characteristic time of ohmic diffusion, and $M_\odot$ denotes the Solar mass.

that is relevant here is the $\alpha\omega$ dynamo, which relies on a combination of nonuniform, Keplerian rotation and helical turbulence to regenerate a magnetic field.

Accretion disks are not all alike, even if we limit ourselves to geometrically thin, optically thick disks. Disks around compact stars such as white dwarfs, neutron stars, and black holes, are relatively small, hot, and well ionized, whereas disks around very young, low-mass stars (stars that are assumed to become Sun-like stars) are relatively large, cool, and, for the most part, weakly ionized. Table 1 shows a comparison of typical parameters of both classes of accretion disks.

For a typical accretion disk around a compact star the value of the magnetic Reynolds number R_m is higher than values of R_m for the Earth's liquid core, the Solar convection zone, or even the galactic gaseous disk. Such a high value of R_m is achieved despite a very small characteristic length scale (taken here to be equal to the disk thickness), and is due to the fast rotational velocity of highly ionized gas. The magnitude of R_m in protoplanetary disks is, *on average*, about as high as in the Earth's core, despite very low ionization levels. This is due to the large characteristic length scale (of the order of 1 A.U.) and fast, Keplerian motion. Thus, the magnetic field and the gas are closely coupled *everywhere* in an accretion disk around a compact star, and at least *somewhere* in a protoplanetary disk, pointing to the possibility that a disk's magnetic field can *in principle* be maintained by an MHD dynamo for as long as there is adequate turbulent flow in the disk.

2 THE ISSUE OF TURBULENCE AND ANGULAR MOMENTUM TRANSPORT IN ACCRETION DISKS

One of the main shortcomings of accretion disk theory is the uncertainty as to the nature of the angular momentum transport responsible for accretion of the mass onto the central object. It is usually assumed that accretion disks are turbulent. This assumption is based primarily on the very large value of their Reynolds numbers, which are as high as 10^{14} for disks around compact stars. The question, however, remains: what is the source of the turbulence? The most obvious candidate is the differential rotation. It has been largely disregarded as a possible source of turbulence because the Keplerian rotation shear is stable with respect to *linear*, infinitesimal perturbation. However, it may be unstable with respect to *nonlinear*, finite amplitude perturbations. Another candidate is convection driven by a superadiabatic temperature gradient across the disk. Again, it is uncertain whether such a gradient can be maintained throughout the significant portion of the disk. Other, more exotic sources of disk turbulence are also highly controversial.

These uncertainties led recently to attempts to construct a disk magnetic dynamo model that does not depend on the existence of turbulent flow in a disk (Vishniac *et al.* 1990; Tout & Pringle 1992). The rationale for such work is that if the successful, *nonturbulent* dynamo can operate in a disk, then a generated magnetic field will provide the necessary angular momentum transport, thus making the issue of turbulence mostly irrelevant to accretion disk theory. In their most recent paper, Tout & Pringle (1992) have proposed a disk magnetic dynamo, in which they invoke the Balbus–Hawley (BH) and Parker instabilities instead of turbulence to close the dynamo loop. This approach capitalizes on the attention that has recently been drawn (Balbus & Hawley 1991) to an instability present in any cylindrical shear flow with an arbitrarily small vertical component of the magnetic field and for which $\partial\omega/\partial r < 0$, where ω is the angular velocity and r the radial coordinate. If present in astrophysical disks (for the argument that BH instability may in fact be absent whenever an azimuthal magnetic field is present see Knobloch 1992), this instability would provide a powerful mechanism of magnetic field amplification, thus making the studies of turbulent disk dynamos mostly academic. Assuming that the BH instability exists, in what accretion disks can it can operate? The BH instability will be damped by a sufficiently high resistive diffusivity η. The condition that damping is unimportant is $V_A^2 \gg 3\omega\eta$, where V_A is the Alfvén speed. On the other hand, there exists a critical wavelength in the BH instability, below which the instability is suppressed. Clearly, this critical wavelength must be smaller than the disk thickness, leading to the condition $V_A < \sqrt{6}c_s/\pi$, where c_s is the speed of sound. These two conditions must be met *simultaneously* in order for the BH instability to exist. In addition, the resulting values of the magnetic field must be such that magnetic pressure is smaller than gas pressure, otherwise the assumption of a thin disk is violated. For an accretion disk around a typical compact star those conditions are met, and the BH instability may in fact provide the most important mechanism of magnetic field generation. In a protoplanetary disk, however, those conditions are irreconcilable except very close to the star, leaving a turbulent dynamo as the leading mechanism of magnetic field regeneration. We consider turbulent dynamos for both types of disk, keeping in mind that in the case of a highly ionized disk it may not be a dominant mechanism of regeneration.

Following Shakura & Sunyaev (1973) we will encapsulate our ignorance about the nature of turbulence into a single parameter $\alpha_\nu = (l_0/h)(v_0/c_s)$, where h is disk half-thickness, l_0 and v_0 are turbulent mixing length and turbulent velocity, respectively. Shakura *et al.* (1978) suggested that irrespective of the source of turbulence, $v_0 \approx \omega l_0$, so $l_0/h \approx v_0/c_s = M_t$, where M_t is the turbulent Mach number. Thus, in our considerations we will take $\alpha_\nu = M_t^2$. The magnetic diffusivity η_t is taken to be identical to the general turbulent diffusion coefficient for a scalar field and equal to $l_0 v_0$, or $\eta_t = M_t^2 h^2 \omega$. The helicity of turbulence, α, is taken on the basis of qualitative order of magnitude arguments given by Ruzmaikin *et al.* (1988) to be equal to $M_t^2 z\omega$, where z is a vertical coordinate. These two statistical parameters η_t and α, together with α_ν, approximate the largely unknown physics of turbulence in an accretion disk. They do not reflect any specific origin of turbulence, nor the influence of rotation on it. In obtaining those parameters, the thin disk approximation $c_s \approx h\omega$ has been explicitly used.

3 COMPUTATIONAL METHOD

Solving the dynamo equation for disk models presents us with many cumbersome mathematical difficulties. First, difficulties arise in handling the boundary condition at infinity in the cylindrical disk geometry. Close to the disk faces, the potential that describes the magnetic field outside the disk interior decays exponentially with distance from the disk surface. On the other hand, far away from a disk, the magnetic field generated by currents in the disk should resemble a multipole and decay as $r^{-(n+1)}$, where $n \geq 2$. We would consider only the approximate boundary conditions: toroidal field and vertical derivative of poloidal vector potential vanish at the disk surfaces. Using such an approximation has only a marginal effect on the character of the generated field inside the disk, and thus on the magnetic angular momentum transport – the major incentive for studying disk dynamos. Even with such simplified boundary conditions, a straightforward numerical solution to the dynamo equation is technically unrealistic due to the tremendous amount of computation required. The practical way of solving the dynamo equation for accretion disks is to take advantage of the great difference between vertical and horizontal dimensions in such disks (see Table 1) and to adopt asymptotic methods. Ruzmaikin *et al.* (1985) pointed out that the so-called adiabatic approximation can be applied to solve the disk dynamo. Stepinski & Levy (1991) formulated a more general and rigorous algorithm based on an idea of adiabatic approximation, which can be directly applied to solving the dynamo equation in the context of an accretion disk. The basic idea is that in the zeroth approximation, when magnetic field diffusion along the disk is considered to be negligibly small as compared with diffusion across the disk, the radial derivatives of the magnetic field can be neglected. The problem thus becomes 'local', with radial coordinate r entering the generation equation only parametrically through radially varying coefficient $D_{eff} = \alpha(r\partial\omega/\partial r)h^3/(\eta + \eta_t)^2$, where η is the resistive magnetic diffusivity. Solving the dynamo equation in this zeroth approximation yields γ, the local rate of magnetic field exponential growth. The zeroth approximation solutions describe the vertical distribution of the magnetic field for fixed or infinitely slow (adiabatic) changes of radial coordinate. The radial structure and the global growth rate Γ of the magnetic field are then determined in the first approximation by the solution of the radial equation with the local growth rate γ used as a 'potential' function subject to the boundary conditions at the inner and outer disk edges. Adiabatic approximation thus provides a way to obtain critical dynamo numbers and radial, as well as vertical, distributions of magnetic field normal modes for accretion disk dynamo described by appropriate α, ω, h, η and η_t.

4 DISKS AROUND COMPACT STARS

Let's consider, as an example, an accretion disk around a black hole with mass $M = 10M_\odot$ (the mass inferred for the black hole candidate Cyg X-1). The inner radius of such a disk is at $3R_g \approx 10^7$ cm, and the outer radius is at about $10^3R_g \approx 10^{10}$ cm (here R_g is the Schwarzschild radius). Taking $M_t \approx 0.1$ and assuming that the mean opacity is well approximated by Kramers' law, the disk temperature is given by $T \approx 6.2 \times 10^4(r/10^{10}\text{cm})^{-3/4}$, and it changes from about 10^7 K at the inner edge to 6×10^4 K at the outer edge. For those temperatures the magnetic resistive diffusivity changes from 10^2 to 10^5 in cgs units. On the other hand, the turbulent diffusivity η_t changes from 10^{14} to 10^{16} in cgs units. Clearly, the magnetic diffusion in such a disk is totally dominated by turbulent diffusion. The radially varying local dynamo number D_{eff} becomes radially constant and equal to $-1.5M_t^{-2} = -1.5/\alpha_\nu \approx -150$. Since the local critical dynamo number is about -12 (Stepinski & Levy 1991), a magnetic field can be maintained by the turbulent dynamo everywhere in a disk around a compact star.

What is the radial structure of the magnetic field in such a disk? Despite the fact that in those disks the local dynamo number remains constant along the disk, the local growth rate, $\gamma(r)$, does change with the radius, and is proportional to ω. Stepinski & Levy (1991) have shown that in a disk where $\gamma(r) \sim \omega \sim r^{-3/2}$ normal modes are localized in the inner parts of the disk. In a disk with $D_{eff} = -150$, a very large number of growing modes will be excited. The fastest-growing mode is confined to the innermost parts of the disk; other progressively less overcritical modes are confined between the inner edge of the disk and some cutoff radius, beyond which the mode is evanescent. The least overcritical mode spans the entire disk. Although those conclusions are obtained in the kinematic limit, the implications for dynamical behavior are clear – the magnetic field is best described as a set of many quasilocalized states each evolving separately from the others. Does magnetic stress dominate viscous stress and thus drive an accretion? The dominant component of the magnetic stress tensor is $t^m_{\phi r} = B_\phi B_r/4\pi$. Assuming that the field intensity is given by the balance of Coriolis and Lorentz forces, $t^m_{\phi r}$ is of the same order as the viscous stress. However, at magnetic Reynolds numbers about 10^{10}, the magnitude of small-scale, random fields can exceed the magnitude of the mean field, and magnetic stress due to the small-scale fields can completely dominate other stresses and control the underlying disk structure.

5 DISKS AROUND YOUNG STARS

The typical steady-state accretion disk around a $1M_\odot$ T Tauri star extends approximately from the star's surface to about 100 A.U. and is parameterized by $\alpha_\nu \approx 0.01$ and an accretion rate of about $10^{-6}M_\odot$ per year. At disk locations where the temperature is above about 200 K, the opacity is dominated by grains such as silicate and Fe metal grains, whereas water ice provides the dominant opacity at locations with lower temperatures. In general, the temperature of a disk is above 1000 K from the inner edge of the disk up to a radius of about 1 A.U., but, since the temperature decreases as $r^{-3/4}$, the extended parts of the disk are indeed too cool to ionize the disk's gas thermally. Stepinski (1992) calculated the ionization state of a protoplanetary disk on the basis that cosmic rays and radioactive nuclei are the dominant sources of ionization. According to those calculations the disk's resistive diffusivity η is about 10^9 (cgs units) in the innermost parts of a disk, but increases to about 10^{17} at a radius of about 1 A.U. where it achieves the maximum. From 1 A.U. outward η slowly decreases to a value of about 10^{16} in the outermost disk. On the other hand, the turbulent diffusivity η_t changes from about 10^{14} in the inner disk to about 10^{15}–10^{16} in the outer disk. Thus, with the exception of the disk's innermost part, magnetic diffusion is dominated by resistive losses. The local dynamo number D_{eff} is not radially constant, as was the case in the disk where magnetic diffusion was dominated by turbulence. Instead it varies radially, falling below its critical value in the intermediate part of a disk, which typically extends from about 2 to 5 A.U. This suggests that a turbulent dynamo operating in an accretion disk around the young star can maintain a magnetic field in the inner and outer regions, but not in the intermediate region of such a disk. The radial structure of the generated field is again likely to consist of a large number of quasilocalized states, because in both inner and outer parts of a disk, where the magnetic field is generated, its growth rate changes steeply with radius. With a moderately large magnetic Reynolds number, the magnitude of small-scale, random magnetic fields and large-scale, average fields is about the same. The magnetic stress is the major mechanism of angular momentum transport, but not so overwhelmingly as was the case in disks around compact stars. In significant portions of the outer disk the magnetic field intensity is limited by an ambipolar diffusion on a lower level than would be inferred from the balance of Coriolis and Lorentz forces.

6 CONCLUSIONS

If accretion disks are indeed turbulent, the standard MHD dynamo can, in general, maintain magnetic fields strong enough to be important to the structure and evolution of those disks. In hot disks around black holes, neutron stars, or white dwarfs, a dynamo can operate successfully throughout the entire radial extent of a disk, but its role in magnetic field regeneration may be secondary if the Balbus–Hawley instability is present. Protoplanetary disks are too poor conductors for the BH instability to work, thus a turbulent dynamo seems, at present, the only mechanism capable of maintaining magnetic field there. The poor conductivity of those disks makes it difficult to establish how effective the dynamo process is there and any conclusions depends strongly on estimates of nonthermal ionization levels. The basic structure of all disk dynamo modes is their spatial localization. This suggests that, contrary to stellar or planetary magnetic fields, disk fields may be composed of many quasilocally generated states, which are unlikely to evolve quietly near equilibrium. It is more likely that disk magnetic fields are characterized by highly variable and episodic dynamical behaviors.

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